# CSE 150A 250A
# AI: Probabilistic Methods

Fall 2025

Instructor: Trevor Bonjour

*Slides adapted from previous versions of the course (Prof. Lawrence, Prof. Alvarado, Prof Berg-Kirkpatrick)*

# Probability Review

# Probability in AI

Probability Theory == "How knowledge affects belief" (Poole and Mackworth)

What is the probability that it is raining out?

How much do I believe that it is raining out?

Viewing probability as measuring belief (rather than frequency of events) is known as the Bayesian view of probability (as opposed to the frequentist view).

# Discrete Random Variables

Discrete random variables, denoted with capital letters: e.g., $X$

Domain of possible values for a variable, denoted with lowercase letters: e.g., $\{x_1, x_2, x_3, \dots, x_n\}$

Example: Weather $W$ ; $\{w_1 = sunny, w_2 = cloudy\}$

# Unconditional (prior) Probability

$$P(X = x)$$

e.g., What is the probability that the weather is sunny?

$$P(W = w_1)$$

# Axioms of Probability

$$P(X = x) \geq 0$$

$$\sum_{i=1}^{n} P(X = x_i) = 1$$

$$P(X = x_i \; or \; X = x_j) = P(X = x_i) + P(X = x_j) \; if \; x_i \neq x_j$$

**Mutually Exclusive!**

# Conditional Probability

$$P(X = x_i | Y = y_j)$$

"What is my belief that $X = x_i$ if I already know $Y = y_j$"

Sometimes, knowing Y gives you information about X, i.e., changes your belief in X. In this case X and Y are said to be *dependent*.

$$P(X = x_i | Y = y_j) \neq P(X = x_i)$$

# Webclicker

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Link:
https://webclicker.web.app/

Course code:
**KSALDG**

# Axioms of Conditional Probability

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Which of the following axioms hold for conditional probabilities?

A. $P(X = x_i | Y = y_j) \geq 0$

B. $\sum_i P(X = x_i | Y = y_j) = 1$

C. $\sum_j P(X = x_i | Y = y_j) = 1$

D. *A and B only*

E. *A, B and C*

# Marginal Independence

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$$P(X = x_i | Y = y_j) = P(X = x_i)$$

Sometimes knowing Y does not change your belief in X. In this case, X and Y are said to be *independent*.

$$P(W = w_i | Y = y_j) = P(W = w_i)$$

If $W$ denotes the weather today, For which variable Y is the above statement most likely true?

A. Y = The weather yesterday
B. Y = The day (Mon, Tue…) of the week
C. Y = The temperature

# More independence

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Consider two students Roberto and Sabrina, who both took the same test. Define the following random variables:

R = Roberto aced the test

S = Sabrina aced the test

**Assume both students have **similar** ability and the only deciding factor for acing the test is the **test difficulty**.*

What is the most logical relationship between $P(R = 1)$ and $P(R = 1|S = 1)$?

A. $P(R = 1) = P(R = 1|S = 1)$

B. $P(R = 1) > P(R = 1|S = 1)$

C. $P(R = 1) < P(R = 1|S = 1)$

# Conditional Independence

Course code: **KSALDG**

What if you also know the test was easy (variable T)?

A. $P(R = 1|T = 1) = P(R = 1|T = 1, S = 1)$

B. $P(R = 1|T = 1) > P(R = 1|T = 1, S = 1)$

C. $P(R = 1|T = 1) < P(R = 1|T = 1, S = 1)$

R and S are conditionally independent given T. i.e., if you already know T, knowing S does not give you additional information about R.

# More independence

Consider these random variables:

- **B** = Burglary occurs
- **E** = Earthquake occurs
- **A** = Alarm goes off
- **J** = Jamal calls
- **M** = Maya calls

Assumptions:

- Burglary and earthquake are independent. ($B \perp E$)
- Alarm can be triggered by either burglary or earthquake.
- Jamal and Maya call if they hear the alarm.
- Jamal and Maya do not directly influence each other.

# Cumulative Evidence

If you know there was an earthquake, what happens to your belief about the alarm going off? Does it increase, decrease or stay the same?

$$P(A = 1) \_\_\_ P(A = 1|E = 1)$$

If you know that there was an earthquake **AND** that there was a burglary. What happens to your belief about the alarm going off?

$$P(A = 1|E = 1) \_\_\_ P(A = 1|E = 1, B = 1)$$

# Explaining Away

If the alarm goes off, what happens to your belief about the burglary?

$$P(B = 1) \_\_\_ P(B = 1|A = 1)$$

Now, if the alarm goes off, and you also get to know there was an earthquake – what happens to your belief about the burglary?

$$P(B = 1|A = 1) \_\_\_ P(B = 1|A = 1, E = 1)$$

# Conditional Independence

- If you don’t know the alarm status, does Jamal’s call change your belief about Maya calling?

$$P(M = 1) \_\_\_ P(M = 1|J = 1)$$

- If you know the alarm went off, does Jamal’s call still change your belief about Maya calling?

$$P(M = 1|A = 1) \_\_\_ P(M = 1|A = 1, J = 1)$$

# Independence

Course code: **GJLOWD**

$$P(X = x_i | Y = y_j) \neq P(X = x_i) \Rightarrow$$
$$P(X = x_i | Y = y_j, Z = z_k) \neq P(X = x_i | Z = z_k)\ ?$$

If X and Y are dependent, does this imply X and Y are dependent given Z?

A. Yes
B. No

# Independence

Course code: **GJLOWD**

$$P(X = x_i | Y = y_j) = P(X = x_i) \Rightarrow$$
$$P(X = x_i | Y = y_j, Z = z_k) = P(X = x_i | Z = z_k)\,?$$

If X and Y are independent, does this imply X is independent of Y given Z?

A. Yes
B. No

# Review - Independence

- Marginal Independence
  - $P(X = x_i | Y = y_j) = P(X = x_i)$
- Conditional Independence
  - $P(X = x_i | Y = y_j) \neq P(X = x_i)$
  - $P(X = x_i | Y = y_j, Z = z_k) = P(X = x_i | Z = z_k)$
- Conditional Dependence
  - $P(X = x_i | Y = y_j) = P(X = x_i)$
  - $P(X = x_i | Y = y_j, Z = z_k) \neq P(X = x_i | Z = z_k)$

# Joint Probability

$P(X = x_i, Y = y_j)$

"What is my belief that $X = x_i$ **and that** $Y = y_j$"

# Joint Probability

Course code: **GJLOWD**

*Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice and participated in anti-nuclear demonstrations.*

Which is more probable?

A. Linda is a bank teller.

B. Linda is a bank teller and is active in the feminist movement.

Tversky & Kahneman(1983)

# Joint Probability

Course code: **GJLOWD**

$P(X = x_i, Y = y_j)$

"What is my belief that $X = x_i$ **and that** $Y = y_j$"

Which of the following is always true?

*A.* $P(X = x_i \; or \; Y = y_j) \leq P(X = x_i, Y = y_j)$

*B.* $P(X = x_i \; or \; Y = y_j) \geq P(X = x_i, Y = y_j)$

*C.* $P(X = x_i \; or \; Y = y_j) = P(X = x_i, Y = y_j)$

*D. None of the above.*

# Joint Probability

$P(X = x_i, Y = y_j)$

"What is my belief that $X = x_i$ **and that** $Y = y_j$"

$T \in \{t_1 = hot, t_2 = cool\}$

W $\in \{w_1 = sunny, w_2 = cloudy\}$

Joint Probability Table

| | T = hot | T = cool |
|---|---|---|
| W = sunny | 0.5 | 0.3 |
| W = cloudy | 0.05 | 0.15 |

# Marginalization

$$P(X = x_i) = \sum_j P(X = x_i, Y = y_j)$$

$T \in \{t_1 = hot, t_2 = cool\}$

$\mathrm{W} \in \{w_1 = sunny, w_2 = cloudy\}$

Joint Probability Table

| | T = hot | T = cool |
|---|---|---|
| W = sunny | 0.5 | 0.3 |
| W = cloudy | 0.05 | 0.15 |

What is P(W=...)?

| | |
|---|---|
| W = sunny | |
| W = cloudy | |

# Product Rule

Connects joint probability with conditional probability

$$P(X = x_i, Y = y_j) = P(X = x_i)\, P(Y = y_j | X = x_i)$$

$$P(X = x_i, Y = y_j) =$$

# Product Rule Generalization

$$P\left(X = x_i, Y = y_j, Z = z_k, \ldots\right)$$

$$= P(X = x_i)P\left(Y = y_j | X = x_i\right)P\left(Z = z_k | X = x_i, Y = y_j\right) \ldots$$

This decomposition often makes reasoning easier.

# Shorthand

Implied Universality:

$$P(X,Y) = P(X|Y)P(Y) = P(Y|X)P(X)$$

Implied Assignment:

$$P(x,y,z) = P(X = x, Y = y, Z = z)$$

# Bayes' Rule

$$P(X|Y) = \frac{P(Y|X)P(X)}{P(Y)}$$

Derive Bayes' Rule from the Product Rule.

# Bayes' Rule example

- Let's assume breast cancer affects 1% of all patients
- When a patient has cancer, a mammogram test comes back positive 90% of the time (true positive)
- When a patient does not have cancer, a mammogram test comes back positive 10% of the time (false positive)

A patient has a positive mammogram test. What is the probability the patient has cancer?

Define the following random variables:

T -> Test is positive

C -> Patient has cancer

# Bayes' Rule example

- Let's assume breast cancer affects 1% of all patients -> $P(C=1) = 0.01$
- When a patient has cancer, a mammogram test comes back positive 90% of the time (true positive) -> $P(T=1|C=1) = 0.9$
- When a patient does not have cancer, a mammogram test comes back positive 10% of the time (false positive) -> $P(T=1|C=0) = 0.1$

A patient has a positive mammogram test. What is the probability the patient has cancer?

Define the following random variables:

T -> Test is positive

C -> Patient has cancer

# Conditioning on Background Evidence

- Product rule without background evidence: $P(X, Y) = P(Y|X)P(X)$
- Claim: $P(X, Y|E) = P(Y|X, E)P(X|E)$

Prove the conditional version of the product rule. (HW)

# Conditioning on Background Evidence

- Product rule (conditional): $P(X,Y|E) = P(Y|X,E)P(X|E)$
- Bayes' rule (conditional):

$$P(X|Y,E) = \frac{P(Y|X,E)P(X|E)}{P(Y|E)}$$

- Marginalization (conditional):

$$P(X|E) = \sum_{y} P(X,Y = y|E)$$

# That’s all folks!